NOTATION

C, c, dimensional and dimensionless impurity concentrations; C_0, initial concentration; D, impurity-diffusion coefficient; L, γ_R, γ_ρ, parts of integration contour; ℓ_{ij}, a_i, d_i, coefficients in the expansion of the functions $w = v + x^2 + y^2$ and $|f|^2$ in the corresponding series of Eqs. (33) and (34); $m = ut - z$, dimensional coordinate of the moving coordinate system; h, halfwidth of the channel in plane problem; h_1, h_2, coefficients in the quadratic form of the velocity profile; Q, amount of impurity in some cross section of the channel per unit area; Q_1, amount of impurity per unit length in the three-dimensional case; P/ℓ, pressure difference per unit length of channel; p, Laplace-transformation variable; T, η, internal variables; t, time; u, maximum value of the velocity; Z, Y, Cartesian coordinates longitudinal and transverse to the flow; v, liquid velocity in channel; $\xi = \tau - z$, variable in coordinate system moving at velocity u; μ, viscosity of liquid; $\Gamma(x)$, Euler gamma function.

LITERATURE CITED

1. A. V. Lykov, Heat and Mass Transfer (Handbook) [in Russian], Moscow (1978).
2. M. L. Michelsen and J. Villansen, J. Heat Mass Trans., 17, No. 11, 1391-1409 (1974).
3. P. V. Tsoi, Methods of Calculating Individual Heat and Mass-Transfer Problems [in Russian], Moscow (1971).
4. G. Taylor, Proc. R. Soc. Lond., Ser. A, 219, No. 1137, 186-203 (1953).
5. R. Aris, Proc. R. Soc. Lond., Ser. A, 235, No. 1200, 67-77 (1956).
6. M. V. Lur'e and V. I. Maron, Inzh.-Fiz. Zh., 36, No. 5, 847-853 (1979).
7. V. V. Dil'man and A. E. Kronberg, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 81-86 (1984).
8. J. Cole, Perturbation Methods in Applied Mathematics [Russian translation], Moscow (1972).
9. N. N. Lebedev, Special Functions and Their Applications [in Russian], Moscow–Leningrad (1963).
10. M. A. Lavrent'ev and B. V. Shabat, Methods of the Theory of Functions of a Complex Variable [in Russian], Moscow (1973).
11. G. A. Grinberg, Selected Problems of the Mathematical Theory of Electrical and Magnetic Phenomena [in Russian], Moscow–Leningrad (1948).

SOLUTION OF THE CONVERSE THERMAL CONDUCTIVITY PROBLEM WITH CONSIDERATION OF THE PERTURBING INFLUENCE OF THE THERMOCOUPLE

S. L. Balakovskii and É. F. Baranovskii

UDC 536.24

Questions involving the use of an adequate model for temperature measurement in solving converse thermal conductivity problems are considered.

Methods for solution of converse thermal conductivity problems are one of the most promising means for adequate processing of data in thermophysical experiment. At the present time a number of highly effective methods have been developed for solution of such problems [1], although the majority of these can be used only under conditions where the perturbing action of thermocouples on heat propagation in the body under study can be neglected. In many cases of practical importance the effect of thermocouples is quite significant [2-5].

In particular, this is true in the study of processes of casting metals and alloys or in temperature measurements in a cutting instrument where the dimensions of the thermocouple, its insulation, and the channel in which these are located are comparable to the distances to the heat source and the area of the surface upon which it acts. In such cases the temperature sensor must be considered as an independent body with its own thermophysical and geometric characteristics, actively participating in heat exchange with the surrounding object.

Mogilev Branch, Physicotechnical Institute, Academy of Sciences of the Belorussian SSR.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 1, pp. 131-134, January, 1987. Original article submitted November 12, 1985.

 0022-0841/87/5201-0106$12.50

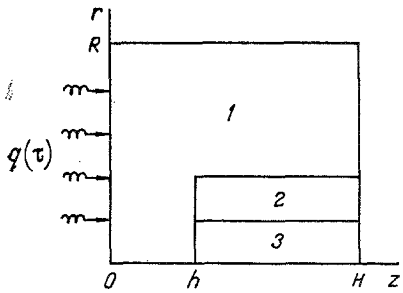

Fig. 1. Region of problem solution: 1) object under study; 2) thermosensor insulation; 3) thermosensor.

In some vicinity around the thermocouple the process of heat propagation in the object (Fig. 1) can be described by the following mathematical model:

$$c(r,\ z)\ \frac{\partial\Theta}{\partial\tau}=\frac{1}{r}\ \frac{\partial}{\partial r}\ \left(r\lambda(r,\ z)\ \frac{\partial\Theta}{\partial r}\right)+\frac{\partial}{\partial z}\ \left(\lambda(r,\ z)\ \frac{\partial\Theta}{\partial z}\right), \tag{1}$$

$$\Theta|_{\tau=0}=0, \tag{2}$$

$$\left.\frac{\partial\Theta}{\partial r}\right|_{r=0}=\left.\frac{\partial\Theta}{\partial r}\right|_{r=R}=\left.\frac{\partial\Theta}{\partial z}\right|_{z=H}=0, \tag{3}$$

$$-\lambda\left.\frac{\partial\Theta}{\partial z}\right|_{z=0}=q(\tau). \tag{4}$$

Assumptions have been made here as to ideal thermal contact between thermocouple-insulation, insulation-wall, and ideal contact between the thermocouple and object is also assumed, which is valid when the thermocouple is attached by capacitive welding. The dimensions of the region to be studied R and H were chosen sufficiently large to eliminate the effect of the thermocouple at the boundaries of the region.

Evaluations of the distortion of the temperature field by the thermocouple were performed for channel and thermocouple dimensions and depth of location with respect to the heated surface characteristic of study of thermal processes in metal and alloy casting in a roller crystallizer [6]. The quantity chosen as a measure of distortion of the temperature field was the deviation of the calculated temperatures ε at the point of the object where the thermocouple would be attached in the presence and absence of the thermocouple.

To determine the factors which most strongly affect the value of ε, a series of calculations was performed for various immersions of the thermocouple from the heated surface h and various thermocouple diameters. The thermocouple material was chromel, while the object to which it was attached was steel 45, with Teflon insulation. In performing the calculations model thermal fluxes were specified, varying over 1 sec from zero to $4\cdot10^{6}$ W/m² linearly.

Results of the calculations permit the conclusion that the depth of thermocouple insertion over the range normally realized in practice, $h = 0.5\cdot10^{-3}-3\cdot10^{-3}$ m, has an insignificant effect on ε. Much more significant is the diameter of the thermoelectrode (see Fig. 2, where data obtained for $h = 10^{-3}$ m and a thermal flux increasing with time as described above are shown). Figure 3 shows the temperature profile at a depth $h = 10^{-3}$ m, corresponding to the thermocouple installation depth 0.25 sec after commencement of heating from the specimen surface. As is evident from the figure, the radius of the thermocouple perturbing action comprises about $2\cdot10^{-3}$ m for a channel radius of $0.6\cdot10^{-3}$ m.

The analysis performed permits the conclusion that in a number of cases there is significant distortion of the temperature fields in objects under study due to the perturbing action of the thermocouple.

Therefore, in order to obtain adequate information on thermal fluxes $q(\tau)$ from thermocouple indications $f(\tau)$ we propose use of a model of heat propagation in the specimen under study which includes heat exchange with the thermocouple.

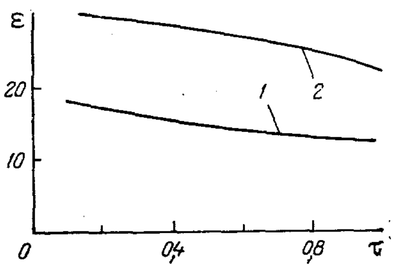

Fig. 2

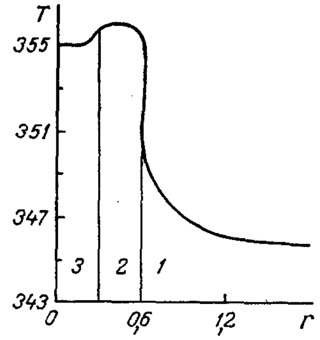

Fig. 3

Fig. 2. Distortion of temperature field vs thermoelectrode diameter d: 1) d = $1.2 \cdot 10^{-3}$ m; 2) $3 \cdot 10^{-3}$. ε, %; τ, sec.

Fig. 3. Temperature profile in thermocouple vicinity: 1) object of study; 2) thermocouple insulation; 3) thermocouple. $r \cdot 10^{-3}$, m; T, °K.

A solution of the problem of reconstruction of the function $q(\tau) = -\lambda(\partial\Theta/\partial z)|_{z=0}$ was performed in extremal formulation by minimizing the functional

$$J(q) = \int_0^{\tau_m} [\Theta(q, 0, h, \tau) - f(\tau)]^2 \, d\tau \tag{5}$$

by the conjugate gradient method.

To calculate Θ a boundary problem with consideration of specimen heat exchange with the thermocouple, Eqs. (1)-(4), was used. The gradient of the functional $J'_q(\tau)$ was expressed in terms of the conjugate function $\psi(r, z, \tau)$ which was calculated from a solution of the boundary problem, obtained in turn from the condition ΔL = 0 [1] where the Lagrangian L can be expressed in the form

$$L = J + 2\pi \int_0^{\tau_m} \int_0^R \int_0^H \psi \left[-c \frac{\partial\Theta}{\partial\tau} + \frac{1}{r} \frac{\partial}{\partial r} \left(r\lambda \frac{\partial\Theta}{\partial r} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial\Theta}{\partial z} \right) \right] r dz dr d\tau. \tag{6}$$

The problem mathematically conjugate to Eqs. (1)-(4) can be written as follows:

$$-c(r, z) \frac{\partial\psi}{\partial\tau} = \frac{1}{r} \frac{\partial}{\partial r} \left(r\lambda(r, z) \frac{\partial\psi}{\partial r} \right) + \frac{\partial}{\partial z} \left(\lambda(r, z) \frac{\partial\psi}{\partial z} \right) + 2(\Theta(r, z, \tau) - f(\tau)) \, \delta(r) \, \delta(z - h), \tag{7}$$

$$\psi|_{\tau=\tau_m} = 0, \tag{8}$$

$$\left.\frac{\partial\psi}{\partial r}\right|_{r=0} = \left.\frac{\partial\psi}{\partial r}\right|_{r=R} = \left.\frac{\partial\psi}{\partial z}\right|_{z=0} = \left.\frac{\partial\psi}{\partial z}\right|_{z=H} = 0. \tag{9}$$

In this case the expression for calculation of the gradient of the functional has the form

$$J'_q(\tau) = -\int_0^R \psi(r, 0, \tau) \, dr. \tag{10}$$

The iterative descent to the minimum of Eq. (5) is accomplished in accordance with the recommendations of [1] by the conjugate gradient method with zero initial approximation of the unknown function $q(\tau)$.

For a numerical realization the boundary problems were replaced by difference equations, solution of which was performed by the locally one-dimensional method with an implicit scheme. Machine time required for solution of the problem on an ES-1022 computer was 4 min when using a nonuniform spatial grid $N_r \times N_z \times N_\tau = 20 \times 25 \times 30$. Solution of the converse problem, performed iteratively, required about 2 h of machine time. The efficiency of the method developed is shown by Fig. 4, which shows results of reconstruction of the model thermal flux under conditions where the perturbing influence of the thermocouple is quite high. The inaccuracy of the reconstruction does not exceed 3% over practically the entire time interval. The divergence between the reconstructed and actual solutions at times close

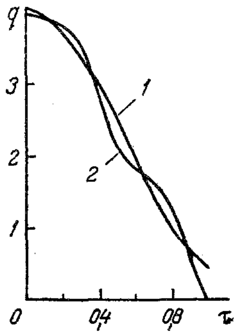

Fig. 4. Solution of the converse thermal conductivity problem: 1) model thermal flux density; 2) reconstructed flux density. $q \cdot 10^{-6}$, W/m^2.

to τ_m can be explained by the fact that in view of the incorrectness of the problem the finite value of the thermal flux density is not refined in the iteration process.

In principle, the method developed here for solution of the converse thermal conductivity problem permits consideration of the perturbing effect of thermosensors of quite arbitrary construction, since change in construction would only lead to a change in formulation of the boundary problem of the type typified by Eqs. (1)-(4).

NOTATION

r, z, radial and axial corodinates; c, λ, specific heat and thermal conductivity; Θ, temperature; $q(\tau)$, thermal flux density; τ, current time; τ_m, measurement time; $f(\tau)$, input temperature; ψ, conjugate function; J, functional; L, Lagrangian; δ, Dirac delta function.

LITERATURE CITED

1. O. M. Alifanov, Identification of Heat-Exchange Processes in Airplane Equipment [in Russian], Moscow (1979).
2. N. R. Kel'tnen and J. V. Beck, Teploperedacha, 108, No. 2, 98-105 (1983).
3. M. V. Kulakov and B. I. Makarov, Measurement of Surface Temperatures of Solids [in Russian], Moscow (1979).
4. N. A. Yaryshev, Izv. Vyssh. Uchebn. Zaved., Priborostr., 6, No. 1, 134-141 (1963).
5. V. N. Deshkin and P. L. Magidei, Energomashinostroenie, No. 5, 13-16 (1961).
6. É. F. Baranovskii, V. M. Il'yushenko, A. A. Stepanenko, and V. N. Tyulyukin, Tsvetn. Met., No. 5, 75-77 (1980).